International Journal of Theoretical Physics, Vol. 40, No. 12, December 2001 (© 2001)

Toward a Collective Treatment of Quantum Gravitational Interactions

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Received March 10, 2001

In this paper we study the gravitational effects induced by the quantum fluctuations of the energy–momentum tensor of scalar fields. Our treatment is based on the two-point correlation function of this operator. In a large N limit, this treatment constitutes the next contribution after the semiclassical treatment. The specific example we study are the gravitational interactions between outgoing configurations giving rise to Hawking radiation and in-falling configurations. Even when the latter are in vacuum state, the interactions grow boundlessly upon approaching the horizon. Their main effect is to wash out the trans-Planckian correlations which existed in a given background geometry. When evaluated in the lowest order, these interactions express themselves in terms of a stochastic ensemble of metric fluctuations. The propagation of Hawking radiation in this ensemble resembles that of sound propagation in a random medium. The analogies with acoustic black holes are manifest even though certain features differ.

1. OVERVIEW

This paper reports on work in progress. We therefore wish to apologize for the lack of clarity and/or completeness which might be found in several places. Our aim is to describe the effects of the gravitational interactions occurring in vacuum. This requires to take into account the quantum fluctuations of the energy momentum tensor of matter fields. This will be done approximatively through the two-point function of the energy–momentum, $\langle T_{\mu\nu}(x)T_{\alpha\beta}(x')\rangle$. We shall show that this treatment is the natural extension of the semiclassical approximation wherein only the one-point function, the expectation value $\langle T_{\mu\nu}(x)\rangle$, is used in Einstein's equations.

When applied to the Hawking radiation, the main virtue of this extension is to wash out the trans-Planckian correlations which existed in the semiclassical treatment without affecting the asymptotic properties of Hawking radiation. More precisely we shall obtain the following results.

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(I) When propagated backwards in time, outgoing quanta are scattered by the metric fluctuations induced by in-falling matter fields in their vacuum state.

(II) These interactions grow so strongly near the horizon that these outgoing quanta are completely scattered. That is, their state becomes completely entangled to that of in-falling configurations.

(III) The in-falling vacuum fluctuations act as a reservoir of modes. This allows for a description of the interactions in terms of a stochastic ensemble of metric fluctuations.

(IV) Even though the spectrum of the latter contains all frequencies (up to a UV cutoff), their impact on outgoing configurations is governed by frequencies $\omega \simeq \kappa$ where κ is the surface gravity of the hole.

(V) The stationarity of vacuum (i.e., the fact that the Green function is a function of the difference $t - t'$ only) leads to stationary metric fluctuations and this, combined with the stationarity of the background metric, gives an effective spread to the horizon which is independent of t.

2. INTRODUCTION

In his original derivation, Hawking (1975) considered the propagation of the radiation in a *given* background metric, that of a collapsing star. This means that the metric is once for all determined by the energy of the collapsing star and is therefore unaffected by the quantum processes under examination. In this approximation, the radiation field satisfies a linear equation (in the absence of matter interactions). One then finds that the in-falling and outgoing field configurations are completely *uncorrelated* near the black hole horizon. In fact the pairs of quanta generated by its formation are composed of two outgoing quanta, one of each side of it. The external ones form the asymptotic flux whereas their partners fall towards the singularity at $r = 0$. Upon tracing over these inner configurations one gets an outgoing incoherent flux described by a thermal density matrix. There is nevertheless a precise relationship between the expectation values of the in-falling and the outgoing energy fluxes. Indeed, the asymptotic outgoing null[2] flux $\langle T_{uu}(r = \infty)\rangle$ is accompanied by a negative in-falling flux $\langle T_{vv}\rangle$ which has, on the horizon $r = 2M$, exactly the opposite value when one works, as we shall do, in the vacuum, that is, $\langle T_{vv}(r = \infty)\rangle = 0$. This follows from the conservation of the radial flux $\langle T_{uu}(r)\rangle - \langle T_{vv}(r)\rangle$ in the static metric outside the collapsing body as well as from the fact that $\langle T_{uu}\rangle$ vanishes like $(r - 2M)^2$ when approaching the future horizon (Davies *et al.*, 1976).

[2] The null coordinates v and u are given by $v = t + r^*$, $u = t - r^*$, and $r^* = r + 2M\ \ln(r/2M - 1)$ is the tortoise coordinate.

This last property becomes crucial when considering the backreaction due these mean fluxes (i.e., the metric change determined by Einstein's equations driven by $\langle T_{\mu\nu}\rangle$). Let us first describe what happens at spatial infinity. There, energy conservation and the hypothesis of adiabaticity,[3] that is, $dM/dt \ll 1$, imply that, in vacuum, the mass loss will be determined by $\langle T_{uu}(r = \infty)\rangle$, thereby describing a geometry characterized by a decreasing Bondi mass. However, to validate the hypothesis of adiabaticity, a local analysis of the evaporating black hole geometry is required. In this analysis, the vanishing of $\langle T_{uu}\rangle$ near $r = 2M$ is crucial. Indeed it is a necessary condition for keeping the regularity of the near horizon geometry during the evaporation process (Barden, 1981; Brout *et al.*, 1995a; Massar, 1995; Parentani and Piran, 1994). Concomitantly, one finds that it is the negative $\langle T_{vv}\rangle$ which drives locally (i.e., $|r - 2M| \ll 2M$) the shrinking of the horizon area according to

$$\frac{dM}{dv} = \langle T_{vv}\rangle|_{r=r_{\text{horizon}}=2M} \simeq -\frac{1}{M^2}. \tag{1}$$

In this equation $M(v)$ is the function which determines the time dependent location of the apparent horizon and which governs the Vaidya metric

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right) dv^2 + 2dv\, dr + r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{2}$$

In brief, the important (and nontrivial) point is that (2) offers a good approximation of the near horizon geometry precisely because of the regularity of the geometry which is preserved by the vanishing of $\langle T_{uu}(r = 2M)\rangle$.

Being regular, this description (known as the semiclassical scenario, a rather imprecise denomination which nevertheless indicates that only mean fluxes are taken into account) would be perfectly valid if another feature of black hole physics wasn't present, namely the field configurations giving rise to Hawking quanta possess arbitrary high (trans-Planckian) frequencies near the horizon: when measured by in-falling observers at r, the frequency grows as

$$\omega \propto \frac{\lambda}{1 - 2M/r} \tag{3}$$

where λ is the asymptotic energy of the quantum. This implies that a wave packet centered along the null outgoing geodesic $u = t - r^*$ had a frequency $\omega \propto \lambda\, e^{\kappa u}$ when it emerged from the collapsing star. ($\kappa = 1/4M$ is the surface gravity and fixes Hawking temperature $T_{\text{H}} = \kappa/2\pi$.) Unlike processes characterized by a typical energy scale, the relation $\omega \propto \lambda\, e^{\kappa u}$ shows that black hole evaporation rests on arbitrary high frequencies. This analysis of wave packets is confirmed by the study of (nondiagonal) matrix elements of $T_{\mu\nu}$. As shown in Massar and Parentani (1996), contrary to the expectation value (the diagonal part) which is regular and

[3] We work with in Planck units: $c = \hbar = M_{\text{Planck}} = 1$.

of the order of M^{-4}, these matrix elements are generically singular on the horizon, that is, their Fourier content is characterized by frequencies ω which grow according to (3).

As emphasized by ’t Hooft (1985), this implies that the gravitational interactions between the configurations giving rise to Hawking quanta and in-falling quanta cannot be neglected, thereby questioning the validity of the semiclassical description. In questioning this validity, two issues should be distinguished, see Section 3.7 in Brout *et al.* (1995a). First, there is the question of the low frequency $O(\kappa)$ changes which can be measured asymptotically, and second, that of the high frequency modifications of the near horizon physics. Since all thermodynamical reasonings indicate that the asymptotic properties (namely thermality governed by κ and stationarity) should be preserved, the problem is to conciliate their stability with the radical change of the near horizon physics which is needed to cure the trans-Planckian problem. Indeed, a perturbative analysis (Parentani, 1999) indicates that near horizon interactions lead to recoil effects which grow like ω in (3). This seems incompatible with the stationarity of the flux.

A new point of view to this problem is provided by the analogy with condensed matter physics pointed out by Unruh (1981) (and revisited by Jacobson (1991, 1993). He noticed that sound propagation in a moving fluid obeys a d’Alembertian equation which defines an acoustic metric. Therefore, when the acoustic metric corresponds to that of a collapsing star thermally distributed phonons should be emitted. However, contrary to photons the dispersion relation of phonons is not linear for frequencies (measured in the rest frame of the fluid) higher than a critical ω_c. Nevertheless, when $\omega_c \gg \kappa$, Unruh (1995) showed that the asymptotic properties of Hawking phonons are unaffected, and this in spite of the fact that frequencies $\omega > \omega_c$ which were solicited in Hawking’s derivation are no longer available. It should also be stressed that the near horizon propagation of the phonon field is very sensitive to the modification of the dispersion relation and drastically differs from that of photons (Brout *et al.*, 1995b; Jacobson, 1996). In brief, the appealing feature of these models is to provide at once, an explanation (in terms of adiabaticity (which essentially follows from scale separation $\omega_c \gg \kappa$, Niemeyer and Parentani, n.d.)) for the stability of the asymptotic properties of the flux, and a simple physical reason (a modified dispersion relation) which eradicates the ultrahigh frequencies.

This is so nice that it strongly suggests that something similar might apply to black holes and solve their trans-Planckian problem. The question is to identify what plays the role of the microscopic constituents of the fluid which introduce, through their interactions, the nontrivial dispersion relation and the cutoff ω_c. As pointed out in Brout *et al.* (1995a) and Jacobson (1996), the natural candidate for this job are the *nonlinearities* induced by gravitational interactions. The main problem one faces is to evaluate the consequences of these interactions. And then, one can offer oneself the luxury to make contact with dumb holes physics by analyzing if/why these effects can be incorporated in a nontrivial dispersion relation, that is, in the *linear* equation governing outgoing radiation.

To answer these two questions clearly requires to go beyond the semiclassical treatment, that is, to take into account the gravitational response due to higher moments of $T_{\mu\nu}(x)$ and not only the classical response driven by the mean $\langle T_{\mu\nu}(x)\rangle$ as in the semiclassical scenario. As a first step towards a full quantum gravitational treatment, we proposed (Parentani, in press) (inspired by Barrabès *et al.*, 2000; Brout *et al.*, 1995a; Casher *et al.*, 1997; Hu and Shiokawa, 1998; Kiem and Verlinde, 1995; Martin and Verdaguer, 2000; Massar and Parentani, 1996) to analyze the gravitational effects driven by the two-point function $\langle T_{\mu\nu}(x)T_{\alpha\beta}(x')\rangle$ still evaluated in the unperturbed vacuum state. This Gaussian treatment gives the lowest order description of the gravitational interactions between outgoing configurations giving rise to Hawking radiation and in-falling vacuum configurations. These interactions lead to collective effects (as in a dilute gas approximation) which express themselves in terms of a stochastic ensemble of metric fluctuations. A simple way to understand this is that the in-falling vacuum configurations (which contain all frequencies up to a UV cutoff) act as an environment for the outgoing quanta. The specification of vacuum state at early times determines the statistical properties of this ensemble and this in turn fixes the cutoff ω_c (in terms of κ) and the frame which breaks the 2D Lorentz invariance (Jacobson, 1991).[4] Then, the main effect of these interactions is to dissipate the trans-Planckian modes near the horizon but without affecting the asymptotic properties of Hawking radiation. Finally, if one wishes, one can represent this dissipation by introducing a phenomenological dispersion relation. The reason is that the dominant gravitational interactions preserve the linearity of the propagation of outgoing configurations since the latter are coupled to in-falling configurations and not to themselves.

The unsolved question concerns the range of validity of this Gaussian treatment. This is a complicate question whose final answer requires a better (full?) understanding of quantum gravity. Let us make a few remarks. First, this question closely follows that concerning the validity range of the semiclassical treatment which is *equally* complicate.[5] Second, our analysis indicates that the semiclassical

[4] In the vicinity of a black hole horizon, there is an induced Lorentz invariance in the u, v plane. This follows from the fact that near the near horizon the 4D d'Alembertian reduces to $\partial_u \partial_v \phi = 0$, see (7), since the mass term and the centrifugal barrier are multiplied by $r - 2M$.

[5] The validity of the semiclassical treatment has been often questioned in rather general terms. However, a significant answer requires to find the physical quantities (i.e., matrix elements of operators) which are erroneously evaluated in this treatment *and* to propose improved expressions for the same quantities in order to see the discrepancy. What is known (Hartle and Horowitz, 1981) is that the semiclassical treatment is the leading contribution in the large N limit when considering N copies with GN held fixed. The important point for us is that the *next* order contribution, that is, the set of graphs weighted by powers of $G^2 N$, corresponds to our treatment. Therefore N is a parameter which organizes the infinity of graphs into series of infinite nonperturbative (in G) subsets whose nth member (i.e., containing powers of $G^n N$) is governed by the nth correlation function of $T_{\mu\nu}$.

treatment will fail before the Gaussian treatment. "Before" should be understood radially, given the blue shift effect encountered during the backward propagation of final configurations specified on $\mathcal{J}^+$, see (3). What emerges is a kind of Russian doll structure in which gravity progressively dominates the physics. Far away from the hole ($r - 2M \gg 2M$) one has outgoing thermal (on shell) radiation. In a first intermediate regime ($\omega_c^{-1} \ll r - 2M \ll 2M$) the modes are still governed by the usual d'Alembertian but observers at fixed r and free falling ones perceive them differently. In the next regime ($\omega_c^{-1} \simeq r - 2M$), inside Jacobson's time-like boundary (Jacobson, 1993), the outgoing modes get severely entangled to the in-falling configuration thereby loosing their "mode" quality. As we shall see, this loss can be described by an effective damping law. Deeper in r, one has some unknown regime governed by Planckian physics. This physics presumably also occurs around us but stays well hidden inside its Planckian husk in the absence of a good microscope.

3. THE MODEL

For simplicity, we shall consider only s-waves propagating in spherically symmetric space times. As in Barrabès (2000), we choose the background metric to be that resulting from the collapse of a null shell of mass M_0 which propagates along $v = 0$. Inside the shell, for $v < 0$, the geometry is Minkowski and described by (2) with $M = 0$. Outside, the metric is also static and given by (2) with $M = M_0$. This choice of the collapsing metric will have no influence in what follows since we shall focus on the vacuum interactions occurring near the horizon.

To identify the various degrees of freedom involved in these interactions, we first analyze the global properties of the massless s-waves in this background. The s-waves fall into two classes according to their support on $\mathcal{J}^-$, the light-like past infinity. The waves in the first class have support only for $v < 0$ and will be noted ϕ_-. They propagate inward in the flat geometry till $r = 0$ where they bounce off and become outgoing configurations. This first class is itself divided in two: for $v < -4M$, the reflected waves cross the in-falling shell with $r > 2M$ and reach the asymptotic region[6] whereas those for $0 > v > -4M$ cross it with $r < 2M$ and propagate in the trapped region till the singularity. The separating light ray $v_H = -4M$ becomes the future horizon $u = \infty$ after bouncing off at $r = 0$. The

This raises the following question: given the dimensionality of $G = l^2_{\mathrm{Planck}}$, can one infer that high orders in n become relevant only for high (Planckian) energies? We conjecture that this is the case: the sorting out of graphs in terms of n is effectively an expansion in the energy of the processes involved in the matrix element under consideration.

[6] $\partial_u \partial_v \phi = 0$ is valid for all r only when working in the geometric optic approximation. In the exact d'Alembertian, see (7), there is a potential around $r = 3M$ which induces partial reflection, a phenomenon irrelevant for our purposes.

configurations that form the second class have support only for $v > 0$ and are noted ϕ_+. They are always in-falling and cross the horizon toward the singularity.

In Hawking's derivation of black hole radiation, owing to the linearity of the field equation, these classical properties also apply in second quantization: the configurations for $v < v_{\rm H}$ give rise to the asymptotic quanta, those for $v_{\rm H} < v < 0$ to their partners (Massar and Parentani, 1996) whereas ϕ_+ plays no role in the asymptotic radiation. One also finds that the correlations between the asymptotic quanta and their partners follow from the fact that, on $\mathcal{J}^-$ and in vacuum, the rescaled field $\phi = \sqrt{4\pi r^2}\chi$ (where χ is the 4D s-wave) satisfies

$$\langle\phi(v)\phi(v')\rangle = \int_0^\infty \frac{d\omega}{4\pi\omega}\, e^{-i\omega(v-v')} = -\frac{1}{4\pi}\, \ln|v - v'|. \tag{4}$$

Since this equation is valid for all v, v' there also exist correlations between ϕ_- and ϕ_+. However, they are physically irrelevant for late Hawking quanta since these emerge from configurations which are characterized by frequencies $\omega = \lambda\, e^{\kappa u} \gg \kappa$ and which are localized extremely close to $v_{\rm H}$. This focusing follows from the asymptotic ($\kappa u \gg 1$) behaviour of the relation between the value of u of the geodesic which originates from v on $\mathcal{J}^-$:

$$V(u) - v_{\rm H} \propto e^{-\kappa\Delta u}. \tag{5}$$

As shown in Hawking (1975), this exponential induces both the thermal radiation at temperature $\kappa/2\pi$ and the necessity of considering trans-Planckian frequencies on $\mathcal{J}^-$. In the absence of gravitational interactions, it also tells us that ϕ_- and ϕ_+ are effectively two independent fields.[7]

This analysis is confirmed by studying the structure of the Fock space of ϕ propagating in the background (2). In this metric, the action of ϕ is

$$S_g = -\int dv\, dr\left[\partial_v\phi\, \partial_r\phi + \frac{1}{2}\left(1 - \frac{2M}{r}\right)(\partial_r\phi)^2\right] \tag{6}$$

with $M(v) = 0$ for $v < 0$ and $M(v) = M_0$ for $v > 0$. Being interested in the near horizon physics, we have dropped the quantum potential term of s-waves, $(2M_0/r^3)\phi^2$, since it does not affect the near horizon propagation. This can be seen by using the double null coordinate system $u = v - 2r^*, v$. Using them, the 4D- d'Alembertian reads

$$\left[\partial_v\partial_v - \left(1 - \frac{2M_0}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M_0}{r^3}\right)\right]\phi_l = 0 \tag{7}$$

where ϕ_l is the rescaled mode of angular momentum l. Thus, as emphasized in Kiem and Verlinde (1995), the propagation of waves (at fixed angular momentum and even for an arbitrary mass) effectively obeys a 2D conformal invariance in

[7] By independent we mean that by sending quanta described by wave packets built with ϕ_+ only, there is no induced emission (Wald, 1976).

the near horizon geometry.[8] This is confirmed by the fact that, classically, the 2D trace of 2D part of $T_{\mu\nu}$ vanishes *off-shell*. Thus, in our model for s-waves, $T_{\mu\nu}$ has only two components, T_{vv} and T_{uu}.

When considering ϕ in second quantization, the 2D conformal invariance implies that the Fock space is a tensorial product of an outgoing u-sector (represented here by ϕ_-) and an in-falling v-sector (represented by ϕ_+). This means that any matrix element of ϕ is expressed in terms of matrix elements of ϕ_- and ϕ_+ which can be evaluated separately. This disconnection into two sectors implies the vanishing of the connected part of the two-point correlation $\langle T_{vv}(x)T_{uu}(x')\rangle_c = \langle T_{vv}(x)T_{uu}(x')\rangle - \langle T_{vv}(x)\rangle\langle T_{uu}(x')\rangle$ for all factorized states (i.e., $|\Psi\rangle = |\Psi_+\rangle \times |\Psi_-\rangle$). This vanishing means that the *fluctuations* of the fluxes around their mean are uncorrelated. Finally, in spite of this absence of correlation, the *mean* value of $T_{vv}(x)$ and T_{uu} are related—through the 2D trace anomaly (Davies *et al.*, 1976) in the present model—as emphasized in Section 2.

4. THE GRAVITATIONAL INTERACTIONS BETWEEN ϕ_- AND ϕ_+

The aim of this section is to describe the gravitational interactions between ϕ_- and ϕ_+. In the next section, we shall compute the consequences of these interactions for Hawking radiation. The generating functional governing the matter-gravity system is

$$Z = \int \mathcal{D}\phi\, \mathcal{D}h\, e^{i[S_{g+h}+S_{h,g}]}. \tag{8}$$

In this equation, h is the change of the metric with respect to the background g discussed above and $S_{h,g}$ is the action of h obtained from the Einstein–Hilbert action. S_{g+h} is the action of ϕ propagating in the fluctuating geometry $g+h$.

When the metric fluctuations are spherically symmetric, h can be characterized by two functions ψ, μ which are completely determined by the energy–momentum tensor of ϕ. The line element in the fluctuating metric can be written as (Barrabès *et al.*, 2000)[9]

$$ds^2 = e^{\psi}\left[-\left(1-\frac{2M}{r}\right)dv^2 + 2dv\,dr\right] + r^2\,d\Omega_2^2 \tag{9}$$

[8] This invariance leads to the trans-Planckian problem: The steady production rate of outgoing quanta arises from an integral over in-frequencies ω whose measure is that of a 2D massless field thereby giving $d\omega/\omega = \kappa\, du$, for more details concerning this equality which follows from (3) see, for example, Parentani (1999) or Eq. (2.54) in Brout *et al.* (1995a).

[9] This line element differs from that used by Bardeen (1981): $ds^2 = e^{\psi}[-e^{\psi}(1-\frac{2M_0+2\mu_B}{r})\,dv^2 + 2dv\,dr] + r^2\,d\Omega_2^2$. The ψ function is the same whereas, to first order in ψ and μ_B, $= \mu = \mu_B - \psi(r-2M_0)/2$. The usefulness of our choice is that ψ no longer affects the null geodesics. We recall that Einstein's equations read $\partial_v\mu_B = T_{vv} - T_{uu}$ and $\partial_{r^*}\psi = 4T_{uu}/(r-2M)$, when expressing $T_{\mu\nu}$ in terms of the two null fluxes T_{vv}, T_{uu}.

where $M = M_0 + \mu(v, r)$ for $v > 0$. In this new metric, the matter action is the same as in Eq. (6):

$$S_{g+h} = -\int dv\, dr \left[\partial_v \phi\, \partial_r \phi + \left(1 - \frac{2M}{r}\right) \frac{(\partial_r \phi)^2}{2} \right]. \tag{10}$$

The new mass function M incorporates the sole change. S_{g+h} is independent of ψ, thereby demonstrating the 2D conformal invariance mentioned earlier.

Our aim is to work out the first order corrections due to the gravitational interactions between ϕ_- and ϕ_+. To this end only quadratic terms in h should be kept in $S_{h,g}$. The Gaussian integration over h can be performed (this is equivalent to solve the linearized Einstein's equations). It gives a self-interacting field theory described by

$$Z = \int \mathcal{D}\phi\, e^{i S_g + i S_{\text{int}}} \tag{11}$$

By construction S_{int} is a quadratic form[10] of the energy–momentum tensor of ϕ.

To identify the various consequences of S_{int} it is most useful to exploit the fact that, when using the free field ϕ, $T_{\mu\nu}$ has only two components, thanks to the 2D conformal invariance. Thus, in a perturbative treatment (such as in the interacting picture) one has to deal with two types of interaction terms only. First one has self-interaction terms depending on ϕ_- or ϕ_+ separately. These terms do not destroy the factorisability of the theory and will not be considered in what follows.[11] One also has a cross term coupling ϕ_- to ϕ_+. The essential point is that this term will inevitably breaks the factorisability of the $\pm$ sectors. Off-shell, the cross term is given by, see (10),

$$S_{\text{int}} = G \int_0^\infty dr \int_0^\infty dv \left[\frac{\mu_+(v, r)}{r} (\partial_r \phi_-)^2 + \frac{\mu_-(v, r)}{r} (\partial_r \phi_+)^2 \right] \tag{12}$$

where $\mu_\pm(v, r)$ is the mass fluctuation driven by $\phi_\pm$ and G is Newton's constant. We have introduced it in the front of μ to read more easily in the equations the

[10] In the t, r coordinate system, that is, when $g_{rt} = 0$, S_{int} is given by the appropriate version (see Eq. (90) in Massar and Parentani (1996) of the so-called BCMN (Berger *et al.*, 1972) Hamiltonian.

[11] When using the free field to evaluate the $\phi_+\phi_+$ contribution to S_{int}, it vanishes on-shell. This can be understood from the fact that the Vaidya metric (2) is an exact solution for any classical infalling massless flux $T_{vv}(v)$. This is confirmed by the fact that the equation of motion of ϕ_+ is still $\partial_r \phi_+ = 0$ even in the presence of self-interactions in the ϕ_+ sector. The $\phi_-\phi_-$ contribution to S_{int} is more tricky to handle in the advanced coordinates v, r. The reason is that infalling geodesics are affected by the presence of an outgoing flux T_{uu} (as clearly seen when using the coordinates u, r). This modification translates in v, r into a deformation of the description of outgoing geodesics $u = u(v, r)$ and it is this effect that is responsible for the $\phi_-\phi_-$ contribution to S_{int}. Let us finally notice that a nonperturbative treatment of the self-interactions of ϕ_- has been developed in Kraus and Wilczek (1995) and Massar and Parentani (2000). It leads to small effects $O(\kappa/M)$ and induces no damping of the waves when approaching the horizon.

order of the interactions between ϕ_- and ϕ_+. On-shell one has $(\partial_r\phi_+)^2 = 0$ and $(\partial_r\phi_-)^2 \simeq (\partial_v\phi_-)^2/(r/2M-1)^2$.

Therefore, the dominant contribution to S_{int} is governed by μ_+ evaluated on the horizon. When ϕ_+ is on-shell it is given by

$$\mu_+(v) = \mu_+(v, r)|_{r=2M} = \int_0^v dv' \, (\partial_{v'}\phi_+)^2. \tag{13}$$

In brief,

$$S_{\text{int}} = G \int_0^\infty dr \int_0^\infty dv \, \frac{\mu_+(v)}{r} (\partial_r\phi_-)^2 \tag{14}$$

represents the dominant $\phi_-\phi_+$ interactions mediated by gravity. Similar expressions have been already considered in by many authors, see, for example, Casher *et al.* (1997), Kiem and Verlinde (1995), and 't Hooft (1985). The novelty of the treatment presented below lies in the treatment of Eq. (14) when the state of ϕ_+ is vacuum.

Before proceeding, let us first relate (11) to Hawking's approach (Hawking, 1975) and the semiclassical treatment. Hawking's approach is recovered by putting $G\mu_+ = 0$. Then Z factorizes as Z_+Z_- (upon ignoring the trace anomaly) and ϕ_- is a free outgoing field propagating in the (fixed) background geometry g. Thus ϕ_+ drops out from all matrix elements built with the operator ϕ_-. It should be emphasized that the trans-Planckian problem (i.e., the fact that matrix elements such as the *in–out* Green function are characterized by trans-Planckian frequencies when one of the operator approaches the horizon, Barrabès *et al.*, 2000; Massar and Parentani, 1996) encountered in Hawking's approach directly follows from this factorisability. Indeed it is the absence of coupling to *other* degrees of freedom which permits the unbounded growth of frequencies upon approaching the horizon.

The semiclassical treatment is generally described by Einstein's equations driven by the mean fluxes. This mean field approach can also be obtained from (11) by splitting $\mathcal{D}\phi$ as $\mathcal{D}\phi_+\mathcal{D}\phi_-$, by integrating *freely* over ϕ_+ (i.e., by ignoring the coupling to ϕ_- upon integrating over ϕ_+), and by retaining only the *mean* $\langle\mu_+(v)\rangle$. This mean change is driven through (13) by the (properly subtracted, Brout *et al.*, 1995a) expectation of $T_{vv} = (\partial_v\phi_+)^2$

$$\langle T_{vv}(v)\rangle|_{r=2M} = -\frac{\pi}{2}\left(\frac{\kappa}{2\pi}\right)^2 \tag{15}$$

evaluated in the unperturbed vacuum (4). This flux has the opposite value of a 2D thermal flux and drives black hole evaporation according to (1). Then, the path integral over ϕ_- is that a free field propagating in the classical metric (2). The only change with respect to the fixed background approach of Hawking is the replacement of M_0 by $M_0 + G\langle\mu_+\rangle$. Therefore the matrix elements of ϕ_-, for example, the Bogoliubov coefficients, are hardly affected (Massar, 1995) by the evaporation as long as it is slow, that is, as long as $M(v) \gg M_{\text{Planck}}$. Therefore, in the semiclassical scenario, the trans-Planckian problem stays as in Hawking's approach: the coupling between ϕ_- and the mean change $\langle \mu_+ \rangle$ is incapable to provide a taming mechanism since it does not open new interacting channels.

To solve this problem clearly requires to take into account the *fluctuating* character of the interactions between ϕ_- and ϕ_+, that is, the possibility of entangling their wave functions. As we shall see, in a perturbative approach, the relevant fluctuations are encoded in the moments of T_{vv} higher than its mean (15) but still evaluated in the unperturbed vacuum (4).

5. THE MODIFIED MATRIX ELEMENTS OF ϕ_-

Our aim is to see how the matrix elements of ϕ_- are affected by their coupling to ϕ_+ in its vacuum state. The computation of these elements requires the integration over ϕ_+ in (11). This integration will determine the influence functional (IF) (Feynman and Hibbs, 1965) governing the effective dynamics of ϕ_-. In spite of the fact that the integration over ϕ_+ is Gaussian it cannot be performed exactly, contrary to that over h in (8). The reason is that the final state of ϕ_+ will be correlated to that of ϕ_-.

However, when computing matrix elements of ϕ_- in the lowest order in G, this entanglement can be neglected, thereby recovering a situation analog to that of h in (8). Indeed the back-reaction effects on these matrix elements which occur through the modification of ϕ_+ are of higher order in G. This approximation concerning degrees of freedom *not* directly involved in the matrix elements (i.e., which factorized out in the absence of interactions) is a common procedure both in quantum field theory where it gives the vacuum contribution, see Chapter 9 in Feynman and Hibbs (1965), and in statistical mechanics (e.g., the *polaron*, Chapter 11). In our case, in this approximation, the IF gives rise to a nonlocal action which is a sum of terms containing $(\partial_r \phi_-)^2$ and kernels given by the Wick contractions of T_{vv} evaluated with (4).[12] The first term is quadratic in $(\partial_r \phi_-)^2$ and the kernel is the (connected) two-point function

$$\langle T_{vv}(v) T_{vv}(v') \rangle_{\mathrm{c}} = \frac{1}{16\pi^2} \frac{1}{(v - v')^4}. \tag{16}$$

[12] If we did not make approximation, the Wick contractions of ϕ_+ would have given rise to a series in G which starts with (16) and with higher order terms depending on ϕ_-. In this case, (16) would have become operator-valued (Kiem and Verlinde, 1995) in ϕ_-, thereby obtaining a situation analog to that of transition amplitudes when enlarging the quantized phase space so as to take into account recoil effects (Massar and Parentani, 1997; Parentani, 1995).

Using (13), one obtains

$$\langle \mu_+(v)\mu_+(v')\rangle = \frac{1}{96\pi^2}\frac{1}{(v-v')^2}$$
$$= \frac{1}{96\pi^2}\int_0^\infty d\omega\,\omega\,\cos[\omega(v-v')]. \tag{17}$$

This equation gives the mean metric fluctuations driven by ϕ_+ in the unperturbed ($G = 0$) vacuum state, see Martin and Verdaguer (2000) for a general study of two-point functions of induced metric fluctuations.

Keeping only this term in the IF is equivalent to work with a stochastic (i.e., a classically given) Gaussian ensemble of metric fluctuations.[13] By equivalent we mean that all matrix elements of operators built with ϕ_- can be computed from this stochastic theory. This possibility occurs precisely because we excluded the correction terms in powers of G which are operator valued in ϕ_-.

In what follows we shall focus on the *in–out* and the *in–in* Green function of ϕ_-. Then, because of the Gaussianity of the stochastic ensemble and the conformal invariance of (6), one can obtain (Barrabès *et al.*, 2000) the nonlinear effects in G from the characteristics of the equation for ϕ_-

$$\left[2\partial_v + \left(1 - \frac{2M_0 + 2G\mu_+(v)}{r}\right)\partial_r\right]\phi_- = 0. \tag{18}$$

These are the outgoing null geodesics $u(v, r)$, solutions of $ds^2 = 0$ of (9). The background solution is $u_0 = v - 2r^*$. The first order change $\delta u = u - u_0$ is determined by a nonhomogeneous equation[14] whose solution is

$$\delta u(v)|_{u_0} = G\int_v^\infty dv'\,\frac{2\mu_+(v')}{r(v')|_{u_0} - 2M_0} \tag{19}$$

where $r(v)|_{u_0}$ is obtained by inverting $u_0(v, r) = v - 2r^*$. The important point for what follows is that the integral in (19) is dominated by the near horizon region where $r(v)|_{u_0} - 2M_0 \simeq 2M_0\, e^{\kappa(v-u)}$. This dependence in $e^{\kappa u}$ will reduce the gravitational effects due to UV part of the spectrum of metric fluctuations.

To determine the physical effects of these fluctuations, let us first analyze the asymptotic plane waves representing Hawking quanta. In the absence of metric

[13] This approximation can also be expressed in terms of classes of Feynman diagrams. To sort them out is useful to consider N copies of ϕ_+. Then the semi-classical treatment consists in keeping all graphs which are weithed by powers of *GN*. This is well known, see, for example, Hartle and Horowitz (1981). Similarly, the quadratic approximation based on (16) consists in keeping graphs weithed by powers of G^2N. This approach based on Feynman diagrams is currently under examination and will be published elsewhere.

[14] This equation is easily obtained from the fact that $2\partial_v + (1 - 2M_0/r)\,\partial_r$ defines $2\partial_v|_{u_0}$ (by definition of the outgoing null geodesics $u_0(v, r) =$ constant). Indeed the first order change in u obeys the nonhomogeneous equation $\partial_v|_{u_0}\delta u = (\mu_+/r)\partial_r|_v u_0$ thereby giving (19).

fluctuations the plane wave $e^{-i\lambda u}$ behaves near the horizon as

$$e^{-i\lambda u_0(v,r)} \simeq \theta(r - 2M_0)\, e^{-i\lambda v}(r - 2M_0)^{i\kappa\lambda}. \tag{20}$$

It vanishes for $r < 2M_0$ and possesses an infinite number of oscillations as $r \to 2M_0$ with increasing momentum $p_r = -i\partial_r$. This is the trans-Planckian problem.

In a Gaussian ensemble of metric fluctuations the dominant—see Appendix A in Barrabès *et al.* (2000)—part of the ensemble average waves is given by

$$\langle e^{-i\lambda u(v,r)}\rangle \simeq e^{-i\lambda u_0(v,r)}\, e^{-\frac{\lambda^2}{2}\langle\delta u(v)\delta u(v)\rangle}. \tag{21}$$

Using (17) and (19), one obtains

$$\begin{aligned}\langle \delta u(v)|_{u_0}\delta u(v)|_{u_0}\rangle &= G^2\int_0^\Lambda \frac{d\omega}{3}\,\frac{\kappa^2\omega}{\kappa^2+\omega^2}\,\frac{1}{(r/2M_0-1)^2}\\ &= \bar{\sigma}_\Lambda^2\,\frac{1}{(r/2M_0-1)^2}\end{aligned} \tag{22}$$

where the spread $\bar{\sigma}_\Lambda$ is equal to $G_\kappa\sqrt{\ln(\Lambda/\kappa)/3}$. We have introduced the UV cutoff Λ to define the integral over ω. Notice that Λ is a Lorentz scalar, since it is the energy of an s-wave in its rest frame. Notice also that its value is hardly relevant since $\bar{\sigma}_{\Lambda=M} = \sqrt{2}\bar{\sigma}_{\Lambda=1}$.

The main result of (22) is that $\bar{\sigma}_\Lambda$ is not proportional to Λ even though $\langle\mu_+^2\rangle \simeq \Lambda^2$. This is because high frequencies ($\omega \gg \kappa$) are damped by the integration over v' in (19). The frequencies $\omega \simeq \kappa$ dominate in (22).

Since $\langle\delta u\,\delta u\rangle$ diverges as $r \to 2M_0$, (21) tells us that the correlations between asymptotic quanta and early configurations, which existed in a given background as shown in (20), are washed out by the metric fluctuations once $r - 2M_0 \simeq \bar{\sigma}_\Lambda \simeq 1/M_0$. The physical reason of this loss of coherence is that the state of ϕ_+ becomes correlated to that of ϕ_- (Kiem and Verlinde, 1995; 't Hooft, 1985). Phenomenologically this loss can be viewed as a dissipation of outgoing waves. Then, as in condensed matter (Jacobson, 1991; Unruh, 1981), it one can be approximatively included in the wave equation through a nontrivial dispersion relation (Barrabès *et al.*, 2000).

We should now explain what is the physical relevance of these results. This is a subtle question. It requires to identify the matrix elements of ϕ_- governed by the ensemble averaged waves (21) and those which aren't. The simplest example of an operator governed by (21) is provided by the *in–out* Green function with one operator at v, r, and the other on $\mathcal{J}^+$. Indeed since the "second" point lives on $\mathcal{J}^+$ where the *out* vacuum was defined, the phase of the out-wave function evaluated at fixed u is not affected by metric fluctuations. On the contrary that of the wave function evaluated near the horizon at v, r is sensitive to the metric fluctuations encountered from $\mathcal{J}^+$ to where it lives.[15] It is this (*unusual*, see ahead) discrepancy in the modification of the phase at each point which explains why the ensemble averaged one-point waves (21) govern this two-point Green function.

It should indeed be emphasized that *usual* expectation values, as for instance the *in–in* Green function with two points evaluated at fixed u on $\mathcal{J}^+$, are *not* governed by the ensemble averaged waves (21). The reason is that the ensemble average is performed after having computed the operator for each member of the ensemble. (This is not a choice: our stochastic classical ensemble is merely a tool to *reproduce* quantum mechanical expectation values. This quantum origin fixes the rules of the ensemble averaging without ambiguity.) In our case, this implies that the shift (19) affects *coherently* the phase at each point (Barrabès *et al.*, 2000). This is important since it guarantees that the shift drops out in the coincidence point limit. This cancellation in turn guarantees that the asymptotic properties are unaffected since the Green function possesses the usual Hadamard singularity (Freedenhagen and Haag, 1990).

We would like to point out that the fact that the metric fluctuation affect very differently these matrix elements can be considered as providing weight to the "complementarity" principle (Kiem and Verlinde, 1995), that is, to the fact that the physics as seen by infalling observers differs from that reconstructed from observers at large distance from the hole. Indeed, the above mentioned difference of the impact of gravitational interactions follows from the fact that asymptotic observers inevitably use *out*-states to probe the physics. Therefore, the overlaps they consider will be automatically of the *in–out* type since the Heisenberg state of the field is specified (prepared) before the collapse. It is this two-states formalism giving rise to nondiagonal matrix elements (Massar and Parentani, 1996) (exactly like in a S-matrix formulation, 't Hooft, 1985) which is at the origin of this difference: the metric fluctuations cannot affect coherently configurations specified in the "ket" on $\mathcal{J}^-$ and in the "bra" on $\mathcal{J}^+$.

6. CONCLUSIONS

We have studied the effects induced by the gravitational interactions governed by (14). Even though we worked out only the lowest order in $G(\bar{\sigma}_\Lambda \propto G)$ we

[15] The cautious reader might wonder if the effects we are describing are not induced by the choice of working at fixed u or at fixed v, r. To waive his qualms, we would like to recall that Green functions have no physical meaning per se, rather they are elements which appear in integrals describing transition amplitudes (for a discussion of this point in a quantum gravitational context see Section 2 in Parentani (1997). It is through this channel that one can verify that u is a physically meaningful coordinate on $\mathcal{J}^+$ since $du|_r = dt$ where dt is the proper time of a particle detector on $\mathcal{J}^+$. Indeed when additional quantum mechanical systems are coupled to the radiation field, the matrix elements governing transition amplitudes will have, in their integrand, phase factors behaving like $e^{-i\lambda u}$ in *any* coordinate system. Similarly, upon questioning what an infalling observer might see when crossing the horizon, v, r are meaningful since $dr|_v \propto d\tau$ where $d\tau$ is the proper time of the free falling guy.

believe that the five results listed in the first section are robust. We see no reason for higher order terms to *suppress* the entanglement of ϕ_- with ϕ_+ so as to give $\bar{\sigma}_\Lambda = 0$ thereby *recovering* trans-Planckian correlations. Indeed the modifications to (16) and (17) should be of the type $(G\omega^2)^n$ or $(\omega/\Lambda)^m$ and therefore will not affect the low frequency behaviour of (17) thereby leaving the effective spread $\bar{\sigma}_\Lambda$ essentially untouched. Moreover, when considering the effects of higher angular momentum modes, as indicating by Casher *et al.* (1997), $\bar{\sigma}$ will be *larger* than our estimate based on s-modes only. In brief, we claim that the entanglement of ϕ_- with ϕ_+ is unavoidable given the fact that gravitational interactions grow with the energy. The entanglement will prevent the unbounded growth of frequencies encountered in the free field theory and will be accompanied by the reorganization of the description of vacuum in terms of free states. By this we mean that the usual states of the free field theory, giving rise to the notion of on-shell particles, provide bad approximations of the true "normal" modes of the interacting theory when approaching the horizon. It is this growing orthogonality as $r \to 2M$ which leads to the dissipative-like behaviour (21).

ACKNOWLEDGMENTS

I wish to thank R. Balbinot, C. Barrabès, R. Brout, L. Ford, V. Frolov, B. L. Hu, T. Jacobson, S. Liberati, S. Massar, E. Verdaguer, and G. Volovik for very useful discussions. I also thank the organizers of the 5th Peyresq meeting on *Quantum Spacetime, Brane Cosmology and Stochastic Effective Theories*, where some of the ideas presented here crystallized, as well as the organizers of the 1st Rio meeting on *Analog Models of General Relativity* for animated discussions.

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